Limits, Continuity, and Derivative Definition

1.

$\lim\limits_{n\to\infty} \frac{4n^2}{n^2+10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

Answer: D

2.

If $f(x)=e^x$, which of the following is equal to $f'(e)$?

(A) $\lim\limits_{h\to 0} \frac{e^{x+h}}{h}$ (B) $\lim\limits_{h\to 0} \frac{e^{x+h}-e^e}{h}$ (C) $\lim\limits_{h\to 0} \frac{e^{e+h}-e}{h}$

(D) $\lim\limits_{h\to 0} \frac{e^{x+h}-1}{h}$ (E) $\lim\limits_{h\to 0} \frac{e^{e+h}-e^e}{h}$

Answer: E

3.

$\lim\limits_{x\to 0}(x\csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

Answer: D

4.

$$\lim_{x\to 3} \frac{x^2+2x-3}{x^2+6x+9}$$

Answer: DNE

5. True or False

If $\lim\limits_{x\to a} f(x)=L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x=a$.

(C) $f(x)$ is defined at $x=a$.

(D) $f(a)=L$

(E) None of the above

Answer: All False

Derivative Rules

1.

If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

Answer: 4/3

2. $y=\dfrac{5x+3}{x^2+4x-2}$ Find $\frac{dy}{dx}$

Answer: $\dfrac{-5x^2-6x-22}{(x^2+4x-2)^2}$

3. Tangent line to $y=x(1-2x)^3$ at x = 1

Answer: y + 1 = -7(x – 1)

4. $y=4\sec(3x)\tan(3x)$ Find $\frac{dy}{dx}$

Answer: $12[sec^3(3x)+\ tan^2(3x)\sec(3x)]$

5. $f(2)=-3,\quad f'(2)=6,\quad h(x)=[f(x)]^3$

Find $h'(2)$

Answer: 162

Implicit Differentiation , e, ln, and related topics

1. $y=\ln(3x^3-2x)$ Find $\frac{dy}{dx}$

Answer: $y'=\dfrac{9x^2-2}{3x^3-2x}$

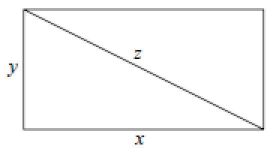
2.

If $3x^2+2xy+y^2=2$, then the value of $\frac{dy}{dx}$ at $x=1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

Answer: E

3.



The sides of the rectangle above increase in such a way that $\frac{dz}{dt}=1$ and $\frac{dx}{dt}=3\frac{dy}{dt}$. At the instant when $x=4$ and $y=3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

Answer: B

4. $y = e^{x^2(lnx)}$ Find $\frac{dy}{dx}$

Answer: $y' = e^{x^2(lnx)}(x + lnx(2x)$

5.

The volume of a cone of radius r and height h is given by $V=\frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

Answer: C

<u>Function Analysis</u>

1.

What is the x-coordinate of the point of inflection on the graph of $y=\frac{1}{3}x^3+5x^2+24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

Answer: D

2.

The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

Answer: A

3.

For what value of k will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

Answer: D

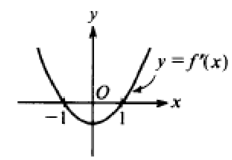
4.

The *derivative* of $f(x)=\frac{x^4}{3}-\frac{x^5}{5}$ attains its maximum value at $x=$

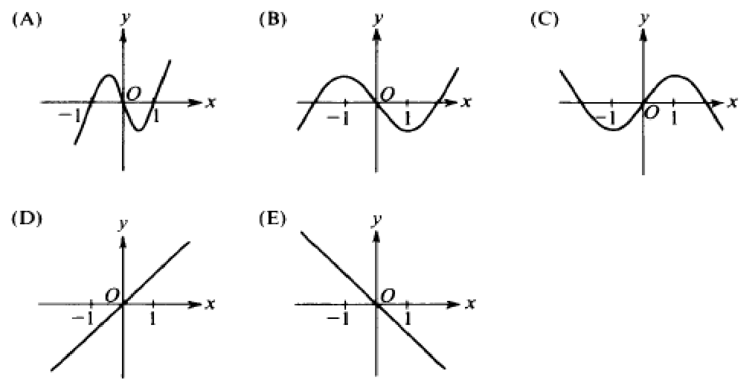
(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

Answer: C

5.



The graph of the <u>derivative</u> of f is shown in the figure above. Which of the following could be the graph of f?



Answer: B

6. $$f(x) = \frac{x^2+1}{x^2-9}$$

Find all critical points. Which of these points is a relative maximum?

Answer: Critical: x = -3, 3, 0 Relative maximum at x = 0